PRECRITICAL CRACK GROWTH DURING CREEP UNDER THE EFFECT OF A VARIABLE LOAD

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Interest in questions of precritical crack growth in metals operating in the high temperature region ($T = (0.4\text{-}0.6)T_m$) has increased in recent decades. These questions are of important practical value for energy machine construction in connection with the possibility of predicting the time of operability of structure elements in which a crack has been detected whose length is considerably less than critical. A large number of experimental studies has been performed devoted to a study of the growth kinetics for such cracks, clarification of the parameters adequately describing the process of their growth. A number of models has been proposed for a theoretical description of the growth process. These papers are covered sufficiently and completely in [1-4]. The absolute majority of authors of the theoretical papers use a damage parameter or another parameter, whose attainment of the critical value defines the time of rupture under creep conditions, for modelling crack growth. The condition of attainment by this parameter of its critical value at the crack apex or at a certain characteristic distance away [6] is taken as the criterion of crack growth [5]. The stress distribution at the crack apex in these papers corresponded to a constant or smoothly varying external load. The problem of crack growth under an abruptly changing load, in particular for a step change in the load that is characteristic for fatigue tests of structures, is examined in this paper.

1. Let us consider the problem of growth of a crack under creep conditions. The governing relationships for such a body with instantaneous elastic deformations taken into account are taken in the form

$$\frac{1}{2}(\dot{u}_{i,j}+\dot{u}_{j,i}) \equiv \dot{\varepsilon}_{ij} = \frac{\dot{\sigma}_{mm}}{9K}\delta_{ij} + \frac{\dot{s}_{ij}}{2G} + \frac{3}{2}B\sigma_e^{n-1}s_{ij}. \tag{1.1}$$

Here u_i are displacement vector components, σ_{ij}, $s_{ij} = \sigma_{ij} - (1/3)\sigma_{mm}\delta_{ij}$ and ε_{ij} are components of the stress tensor, the stress tensor deviator, and the strain tensor, $\sigma_e = ((3/2)s_{ij}\cdot s_{ij})^{1/2}$ is the stress intensity, G and K are the shear modulus and the volume expansion modulus, B, n are creep power-law parameters $\dot{p} = B\sigma^n$, the comma denotes differentiation with respect to the corresponding coordinate, the dot denotes the derivative with respect to time, the subscripts take on the values 1, 2, and 3 summation is over repeated subscripts. To describe the process of rupture that occurs in the body during creep, we use the damage parameter ω with its kinetic equation [7]

$$\dot{\omega} = A\left(\frac{\sigma_{\max}}{1-\omega}\right)^m, \quad \omega(0) = 0. \tag{1.2}$$

From (1.2) with the rupture criterion $\omega(t_p) = 1$ which will be satisfied at the apex of a moving crack in this case and has the form $\omega(l(t), t) = 1$, we obtain an integral equation for the desired dependence $l(t)$ [5]

$$A(m+1)\int_0^t \sigma_{\max}(l(t), \tau)\, d\tau = 1. \tag{1.3}$$

To solve (1.3), the stress distribution must be known in the body with the moving crack.

If the stresses at the crack apex are sought in the form $\sigma_{ij}(r, \theta, t) = r^\lambda f_{ij}(\theta, t)$, then there results from (1.1) that the creep strain rate predominates over the elastic strain rate $\dot{e}_{ij} = (\sigma_{mm}/9K)\delta_{ij} + (s_{ij}/2G)$ as $r \to 0$, and these components can be neglected in comparison with $(3/2)B\sigma_e^{n-1}s_{ij}$. But for the power law $\dot{\varepsilon}_{ij} = (3/2)B\sigma_e^{n-1}s_{ij}$ the singularity index is $\lambda = -1/(n+1)$, therefore, the Hutchinson-Reiss-Rosengren asympototic [8, 9] can be written for $\sigma_{ij}(r, \theta, t)$:

$$\sigma_{ij}(r, \theta, t) = \left(\frac{C(t)}{BI_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}^{(c)}(\theta). \tag{1.4}$$

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Here C(t) is expressed analogously to the J-integral in terms of a contour integral over the contour Γ_ε, that shrinks to the crack apex:

$$C(t) \equiv \lim_{\varepsilon\to 0} \oint\limits_{\Gamma_\varepsilon} \left(\frac{n}{n+1} B\sigma_e^{n+1} \cos\theta - \sigma_{ij} n_j \dot{u}_{i,1}\right) ds = \oint\limits_{\Gamma} \left(\frac{n}{n+1} B\sigma_e^{n+1} \cos\theta - \sigma_{ij} n_j \dot{u}_{i,1}\right) ds + \lim_{\varepsilon\to 0} \int\limits_{V_\varepsilon} \sigma_{ij} \dot{e}_{ij,1} dV. \tag{1.5}$$

The second component in the representation C(t) in terms of a contour integral over the arbitrary contour Γ vanishes in the steady-state creep state ($\dot{\sigma}_{ij} = 0$) while C(t) in this case is none other than the C*-integral of steady-state creep independent of the contour of integration and determined completely by the external conditions of the problem (the size of the body and the crack, the external load).

Taking account of (1.4), the solution (1.3) is easily found and has the form [5]

$$\dot{l}(t) = A\,(m+1) \frac{\pi}{\sin \pi\alpha} \left(\frac{C(t)}{BI_n}\right)^{\alpha} (l - l_0)^{1-\alpha}, \tag{1.6}$$

where $\alpha = m/(n+1) < 1$; l_0 is the initial lenght of the crack. Therefore, if the quantity C(t) is known, then the desired dependence $l(t)$ can be found from (1.6).

2. Upon a sudden loading at the time t = 0 an instantaneous elastic state occurs in the body with stresses whose asymptotic at the apex has the form

$$\sigma_{ij}(r, \theta, t = 0) = \frac{k_I}{\sqrt{2\pi r}}\, \tilde{\sigma}_{ij}^{(e)}(\theta). \tag{2.1}$$

Under constant (t > 0) external loads a stress redistribution occurs from the instantaneous elastic state (2.1) to the steady creep state

$$\sigma_{ij}(r, \theta, t \to \infty) = \left(\frac{C^*}{BI_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}^{(c)}(\theta), \tag{2.2}$$

where $\tilde{\sigma}_{ij}^{(e)}(\theta)$, $\tilde{\sigma}_{ij}^{(c)}(\theta)$ are the circumferential change in stress in the elastic state and in the steady creep state. In the transition period prior to the onset of the steady creep state the stress distribution at the crack apex has the asymptotic (1.4). The expression $C(t) = (1-\nu^2)K_I^2/((n+1)Et)$, that follows from an analysis of the dimensions of the quantities defining C(t) in (1.5) is obtained for the quantity C(t) in [10]. The build-up time for the stationary state t_T is found from condition (2.2) $C(t_T) = C^*$, i.e., $t_T = (t-\nu^2)K_I^2/((n+1)\cdot EC^*)$. Therefore, in this case the following expression can be used for C(t)

$$C(t) = C^* \begin{cases} t_T/t, & 0 < t < t_T, \\ 1, & t \geqslant t_T, \end{cases} \tag{2.3}$$

whose validity is verified in a finite-element solution of the problem [11]. Taking account of (2.3), Eq. (1.6) can be rewritten as

$$\dot{l}(t) = \dot{l}_{CT} \begin{cases} (t_T/t)^{\alpha}, & 0 < t < t_T, \\ 1, & t \geqslant t_T, \end{cases} \tag{2.4}$$

where $\dot{l}_{CT} = A(m+1)\dfrac{\pi}{\sin\pi\alpha}\left(\dfrac{C^*}{BI_n}\right)^{\alpha}(ll_0)^{1-\alpha}$ is the crack growth rate in the steady creep state. The dependence $\dot{l}(C^*)$ is displayed in the figure, where the dependence $\dot{l}_{CT}(C^*)$ is shown by dashes

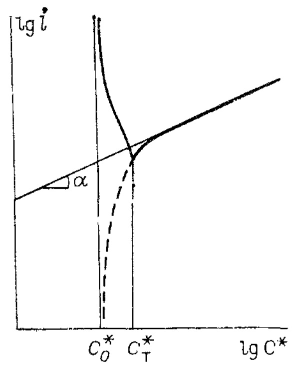

Fig. 1

for $t < t_T$, i.e., $\dot{l}(C^*)$ without taking into account the stress distribution from the instantaneous elastic state to the steady creep state, C_0^* and C_T^* correspond to values of C^* for $t = 0$ and $t = t_T$.

The power-law dependence $\dot{l}(C^*)$ has been observed in a large number of experiments [1-4]. The influence of the stress redistribution in the transient stage (descending branch of the curve $\dot{l}(C^*)$, whose existence is noted in [4]) and the running length of the crack $l(t)$ on the dependence $\dot{l}(t)$ is insignificant and can be estimated as follows. We represent C^* in the form $C^* = B\sigma_\infty^{n+1} l c(l/w, n)$, where σ_∞ is the stress applied to the specimen, w is the specimen width, $c(n, l/w)$ is a dimensionless function of n and l/w whose value is obtained in [12-14] for a number of specimens and certain values of n and l/w. We estimate $C_T^*/C_0^* \simeq l_T/l_0$ (l_T is the crack length for $t = t_T$). Integrating (2.4), we find that $l_T/l_0 \simeq 1 + \frac{c(n, l_0/w)}{I_n}\left(\frac{\pi\alpha}{(1-\alpha)\sin\pi\alpha} t_T/t_p\right)^{1/\alpha}$ ($t_p = \left[A(m+1)\sigma_\infty^m\right]^{-1}$ is the time to rupture of a specimen with mean stress σ_∞ therein). For many materials $t_T/t_p << 1$ (thus, for certain steels $t_T/t_p = 0.01$-0.05 [15]) consequently, the descending branch on the dependence $\dot{l}(C^*)$ is quite often not observed experimentally. The influence of l on $\dot{l}$ also vanishes quite rapidly since $(l - l_0)^{1-\alpha} = ((l - l_0)/(w - l_0))^{1-\alpha}(w - l_0)^{1-\alpha}$ while the quantity $\left(\frac{l - l_0}{w - l_0}\right)^{1-\alpha} \simeq$ for all l, with the exception of $l \sim l_0$, because $1 - \alpha = \frac{n+1-m}{n+1} \simeq \frac{1}{n+1} \ll 1$. Therefore, for $\dot{l}$ under a constant load the following simpler dependence can be used.

$$\dot{l} = A(m+1)\frac{\pi}{\sin\pi\alpha}\left(\frac{C^*}{BI_n}\right)^{\alpha}(w - l_0)^{1-\alpha}, \tag{2.5}$$

which is valid for almost all l, except $l \simeq l_0$.

3. Let us consider cyclic loading of a body when the load applied to it varies as follows $\sigma_\infty^{(1)} \to \sigma_\infty^{(2)} \to \sigma_\infty^{(1)} \to \ldots$. We denote the intervals of operation of $\sigma_\infty^{(1)}$ and $\sigma_\infty^{(2)}$ by t_1 and t_2 and we consider $\sigma_\infty^{(2)} > \sigma_\infty^{(1)} > 0$. At the loading ($\sigma_\infty^{(1)} \to \sigma_\infty^{(2)}$) or unloading ($\sigma_\infty^{(2)} \to \sigma_\infty^{(1)}$) times, the stresses at the crack apex change instantaneously by the quantity $\Delta\sigma_{ij} = \pm\frac{\Delta K_I}{\sqrt{2\pi r}}\tilde{\sigma}_{ij}^{(e)}(\theta)$, as follows from the defining relationships (1.1) written for the stress increments $\Delta\sigma_{ij}$ and the strain increments $\Delta\varepsilon_{ij}$ for an instantaneous change in the external load. The plus and minus signs correspond to loading and unloading, $\Delta K_I = (\sigma_\infty^{(2)} - \sigma_\infty^{(1)})\sqrt{\pi l}\, k(l/w)$, $k(l/w)$ is a dimensionless function of l/w. The total stresses at the time T of the change in the external load have the form

$$\sigma_{ij}(r, \theta, T) = \left(\frac{C(T)}{BI_n r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}^{(c)}(\theta) \pm \frac{\Delta K_I}{\sqrt{2\pi r}}\tilde{\sigma}_{ij}^{(e)}(\theta). \tag{3.1}$$

It follows from (3.1) that as $r \to 0$ the first component can be neglected, i.e., the quantity $\Delta\sigma_{ij}$ completely determines the stress state at the crack apex at the time of a change in the external load. For $t > T$ redistribution starts for the stresses $\Delta\sigma_{ij}$, defining the stress state at the crack apex analogous to (1.4):

$$\Delta\sigma_{ij}(r, \theta, t) = \pm\left(\frac{(1-\nu^2)\Delta K_I^2}{(n+1)EBI_n(t-T)r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}^{(c)}(\theta).$$

The total stress at the crack apex is again determined by (1.4) in which $C(t)$ already depends on $C(T)$ and on $\pm(1 - \nu^2)\Delta K_I^2/((n + 1)E(t - T))$. We approximate the expression for $C(t)$ in the form

$$C(t) = C(T) \pm \frac{(1-\nu^2)\Delta K_I^2}{(n+1)E(t-T)}, \tag{3.2}$$

by taking into account that as $t \to T$ the increment $\Delta C = C(t) - C(T)$ is determined completely by the quantity $\pm(1-\nu^2)\Delta K_I^2/((n + 1)E(t - T))$.

Let the intervals t_1 and t_2 be sufficiently large so that a new stationary state characterized by the quantities C_1^* and C_2^* builds up at the crack apex during this time after a load change. Then according to (3.2), the stress redistribution time to the new stationary state C_2^* can be eliminated for the loading ($\sigma_\infty^{(1)} \to \sigma_\infty^{(2)}$, $C(T) = C_1^*$) as

$$\tau_T = \frac{(1-\nu^2)\,\Delta K_I^2}{(n+1)\,E\left(C_2^* - C_1^*\right)} = \frac{\pi(1-\nu^2)\,k^2}{n+1}\,\frac{e_2}{c\,\dot{p}_2}\,\frac{(1-R)^2}{(1-R^{n+1})}. \tag{3.3}$$

During unloading ($\sigma_\infty^{(2)} \to \sigma_\infty^{(1)}$, $C(T) = C_2^*$) in the time

$$\tau_s = \frac{(1-\nu^2)\,\Delta K_I^2}{(n+1)\,EC_2^*} = \frac{\pi(1-\nu^2)\,k^2}{n+1}\,\frac{e_2}{c\,\dot{p}_2}\,(1-R)^2 \tag{3.4}$$

the stresses are negative, then they reach their new stationary state corresponding to C_1^* during the time τ_T. The $\sigma_\infty^{(1)}/\sigma_\infty^{(2)} = R$ in (3.3) and (3.4) is the cycle asymmetry factor, and $\sigma_\infty^{(2)}/E = e_2$, $B(\sigma_\infty^{(2)})^n = p_2$ are the magnitudes of the instantaneous elastic strain and the creep strain rate in uniaxial tests under the action of stress $\sigma_\infty^{(2)}$. It follows from (3.3) and (3.4) that $\tau_S/\tau_T = (1 - R^{n+1}) \approx 1$ for $R \leqslant 0.5$–0.7 and $n \geqslant 3$, i.e., in this case it can be considered that the stresses are negative during the time τ_T, after which a new stationary state sets in, characterized by the quantity C_1^*. The ratios τ_T/t_1, τ_T/t_2 are quantities on the order of $e_2/\dot{p}_2 t_2$, which for sufficiently large t_2 are small in metals with developed creep strain, i.e., the stress redistribution can be neglected in this case, and it can be considered that $C(t) = C_2^*$ or $C(t) = C_1^*$ almost at once after the loading or unloading time.

For small t_1 and t_2 ($t_1 \approx t_2$), we obtain from (3.2) that the quantity $C(t)$ is negative during the whole interval t_1 for unloading, and $C(t) \approx (1 - \nu^2)\Delta K_I^2/((n + 1)E(t - T))$ for loading, since $C(T)$ is determined by the value of $C(t)$ at the end of the unloading interval, i.e., $C(T) \leqslant 0$. If t_2 is large while t_1 is small, then for loading $C(t) \approx C_2^*$ while for unloading the quantity $C(t)$ will be negative during the whole interval t_1. A quantitative measure of whether t_1 and t_2 are large or small is the ratios τ_T/t_1 and τ_T/t_T (or τ_S/t_1 and τ_S/t_2 for $\tau_S/\tau_T \approx 1$). For $\tau_T/t_1 >> 1$ and $\tau_T/t_2 >> 1$ the quantities t_1 and t_2 are small, while for $\tau_T/t_1 << 1$ and $\tau_T/t_2 << 1$ they are large.

Taking into account all the above on the behavior of the quantity $C(t)$ in the different cyclic loading cases, it can be concluded that for $\tau_T/t_1 << 1$ and $\tau_T/t_2 << 1$ the rate of crack growth $\dot{l}$ is determined by (2.5), where $C^* = C_2^*$ for loading and $C^* = C_1^*$ for unloading.

The increment in the crack length per cycle Δl will be $\Delta l \simeq A(m+1)\frac{\pi}{\sin\pi\alpha}\left(\frac{C_2^*}{BI_n}\right)^\alpha (w - l_0)^{1-\alpha} t_2$ $\left(1 + R^{n+1} t_1/t_2\right)$. In the case when $R^{n+1} \ll 1$, $\Delta l \simeq A(m+1)\frac{\pi}{\sin\pi\alpha}\left(\frac{C_2^*}{BI_n}\right)^\alpha (w - l_0)^{1-\alpha} t_2$, i.e., is determined only by the quantity C_2^* for the loading case. For $\tau_T/t_1 >> 1$ and $\tau_T/t_2 << 1$ the stresses during unloading will be negative, i.e., $\dot{\omega} = 0$ and $\dot{l} = 0$ during the whole unloading interval. Therefore the increment in the crack length per cycle is

$$\Delta l \simeq A(m+1)\frac{\pi}{\sin\pi\alpha}\left(\frac{C_2^*}{BI_n}\right)^\alpha (w - l_0)^{1-\alpha} t_2.$$

The case $\tau_T/t_1 >> 1$ and $\tau_T/t_2 >> 1$ (fatigue crack growth at high temperature) is of special interest. The stresses during unloading are again negagive, i.e., $\dot{l} = 0$, while for the loading interval the quantity $C(t)$ in (2.4) can be considered equal to $C(t) \approx (1-\nu^2)\Delta K_I^2/((n+1)\cdot E(t - T))$. The increment in crack length per cycle

$$\Delta l = A(m+1)\frac{\pi}{\sin\pi\alpha}\left(\frac{(1-\nu^2)\,\Delta K_I^2}{(n+1)\,EBI_n t_2}\right)^\alpha (w - l_0)^{1-\alpha}\frac{t_2}{1-\alpha}$$

can be taken as an estimate of the rate of fatigue crack growth at high temperature dl/dn, i.e.,

$$\frac{dl}{dn} \simeq A(m+1)\frac{\pi}{\sin\pi\alpha}\left(\frac{(1-\nu^2)\,\Delta K_I^2}{(n+1)\,EBI_n t_2}\right)^\alpha (w - l_0)^{1-\alpha}\frac{t_2}{1-\alpha}. \tag{3.5}$$

It follows from (3.5) that the crack growth rate is determined completely by the quantity ΔK_1 in this case.

The fact of crack shutdown at a certain time during partial removal of the load is established experimentally in [16]. It is shown there that for large times t_1 and t_2 the rate of crack growth $\dot{l}$ is determined by the running quantity C^*. It is shown in [17] that for a cyclic change in the load with a small holding time in the cycle t_1 and t_2, $t_1 \approx t_2$, dl/dn

is determined by the quantity $\Delta K_I^{2\alpha}$, while for short-time unloading ($t_1/t_2 << 1$) and large holding time t_2 during loading by the quantity C* as in the case of a constant load.

Therefore, utilization of the damage parameter in bodies with a crack under creep conditions permits prediction of their growth process for both constant and variable loads. Partial removal of the load results in a temporary stop of the crack. The half time τ_S depends on the geometry of the body with the crack (the ratio k^2/c) the loading conditions (the cycle asymmetry factor), and the mechanical properties of the material (the ratio $e_2/\dot{p}_2$). The quantities τ_T/t_1 and τ_T/t_2 that show whether an instantaneous change in the stress influences the crack growth process play an important role. It can be judged from τ_T/t_1 and τ_T/t_2 which parameter (C* or ΔK_T) governs the crack growth process during cyclic loading.

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